

Exercise 8 – 11.12.2025 - Solution

Lateral earth pressure under different hydraulic conditions

Problem statement

A 3-m-high retaining wall is going to be embedded 3 m in a silty soil that extends to considerable depth. To evaluate the horizontal stresses acting on the retaining wall, please answer the following questions.

Question 1 – Hydraulic and mechanical characterization

Provide the hydraulic and mechanical characterization of the soil in saturated and unsaturated conditions by answering the following questions.

- a) The soil has the following properties: dry unit weight $\gamma_d = 16.70 \text{ kN/m}^3$, saturated water content $w_{\text{sat}} = 0.257$. Calculate the saturated unit weight of the soil.
- b) Triaxial tests were conducted and experimental results are available for RTC (Reduced Triaxial Compression) tests. The principal stresses (σ_1 and σ_3) at failure are reported in the following table. Determine the parameters for the Mohr-Coulomb (M-C) failure criterion.

Sample	$\sigma_3 = \sigma_2$ [kPa]	σ_1 [kPa]
1	1	20
2	5	30
3	7	40

- c) Use the Gardner's model to plot the permeability vs. matric suction relationship within the range of matric suction $\leq 10^3 \text{ kPa}$. The following parameters can be assumed: $k_s = 1 \cdot 10^{-6} \text{ m/s}$ and $\alpha = 0.0226 \text{ kPa}^{-1}$.
- d) Use the Van Genuchten's model to plot the Soil Water Retention Curve (SWRC) within the range of matric suction $\leq 10^3 \text{ kPa}$. The following parameters can be assumed: $\alpha = 0.0226 \text{ kPa}^{-1}$, $n = 6.34$, $m = 0.13$.
- e) Based on your answers above, estimate the permeability of the soil when degree of saturation is 50 % and 90 %. Comment on the influence of the degree of saturation on the permeability.

Question 2 – Lateral earth pressure

Draw the matric suction and lateral earth pressure profiles at an active state for the following conditions. Plot the results for depth up to 3 m.

- a) Retained soil is dry (GWT well below the base of the wall)
- b) Retained soil is fully saturated with GWT at the top of the wall
- c) Retained soil is partially saturated due to capillary rise. In particular, the GWT is at the base of the wall). Consider the following scenarios:
 - hydrostatic distribution of the matric suction ($p_a - p_w$)
 - distribution of the matric suction with infiltration rate $q = -1/10 k_s$, $q = -1/5 k_s$, $q = -1/2 k_s$, and $q = -k_s$ being k_s the saturated hydraulic conductivity

Comment on the evolution of the resultant horizontal force acting on the retaining wall when an initially dry retained soil becomes fully saturated (e. g. condition a to b). Also discuss what events/conditions can induce such a change on a site.

Solution

1. Hydraulic and mechanical characterization of the soil in saturated and unsaturated conditions

- a) The definition of dry unit weight and of water content are here recalled:

$$\gamma_d = \frac{W_s}{V}$$

$$w = \frac{W_w}{W_s}$$

in which W_s is the weight of the solid phase, W_w is the weight of the water phase and V is the total volume.

The total unit weight can be computed thanks to the following relationship:

$$\gamma = \frac{W_s + W_w}{V} = \frac{W_s}{V} + \frac{W_w}{W_s} \frac{W_s}{V} = \gamma_d (1 + w)$$

In the case of a saturated soil, we will have:

$$\gamma_{sat} = \gamma_d (1 + w_{sat})$$

Considering the given data:

$$\gamma_{sat} = 21.00 \text{ kN} / \text{m}^3$$

- b) The parameters for the Mohr-Coulomb (M-C) failure criterion can be estimated by plotting the data of the triaxial tests referred to the failure condition in a plane (σ_m , τ_{max})

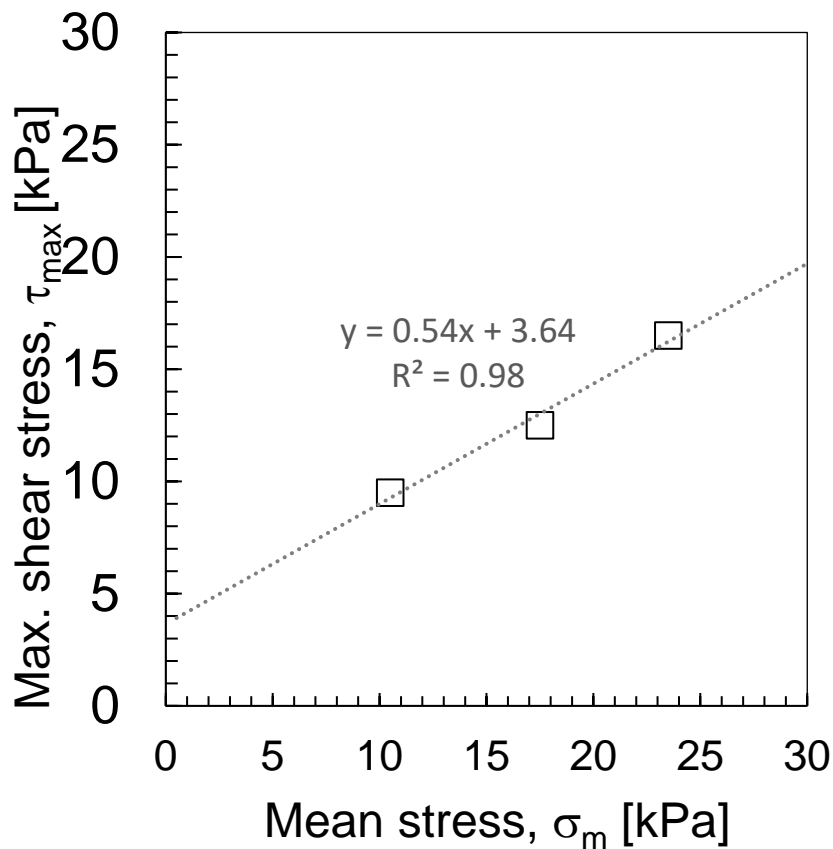
τ_{max} and σ_m can be computed by using the following definitions:

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sigma_m = \frac{\sigma_1 + \sigma_3}{2}$$

Sample	σ_1 [kPa]	σ_2 [kPa]	σ_3 [kPa]	τ_{max} [kPa]	σ_m [kPa]
1	20	1	1	9.5	10.5
2	30	5	5	12.5	17.5
3	40	7	7	16.5	23.5

A linear trendline can be fitted to the experimental points, as shown in the following graph:



The MC failure criterion in the (σ_m , τ_{max}) can be written as follows:

$$\tau_f = c' \cos \varphi' + \sigma_m \sin \varphi'$$

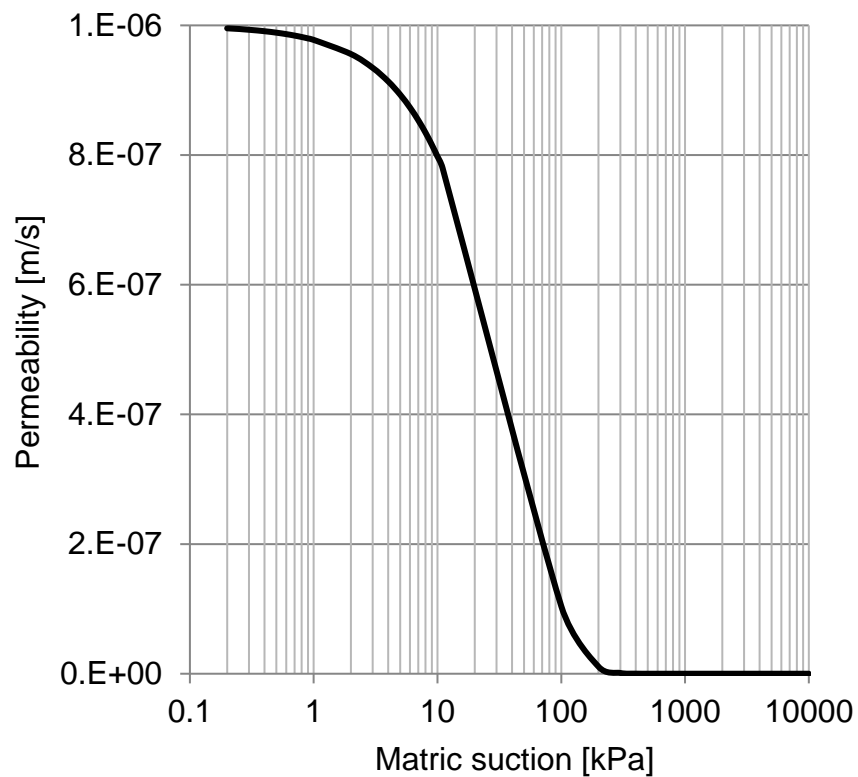
with:

$$c' \cos\varphi' = 3.64 \quad \text{and} \quad \sin\varphi' = 0.53$$

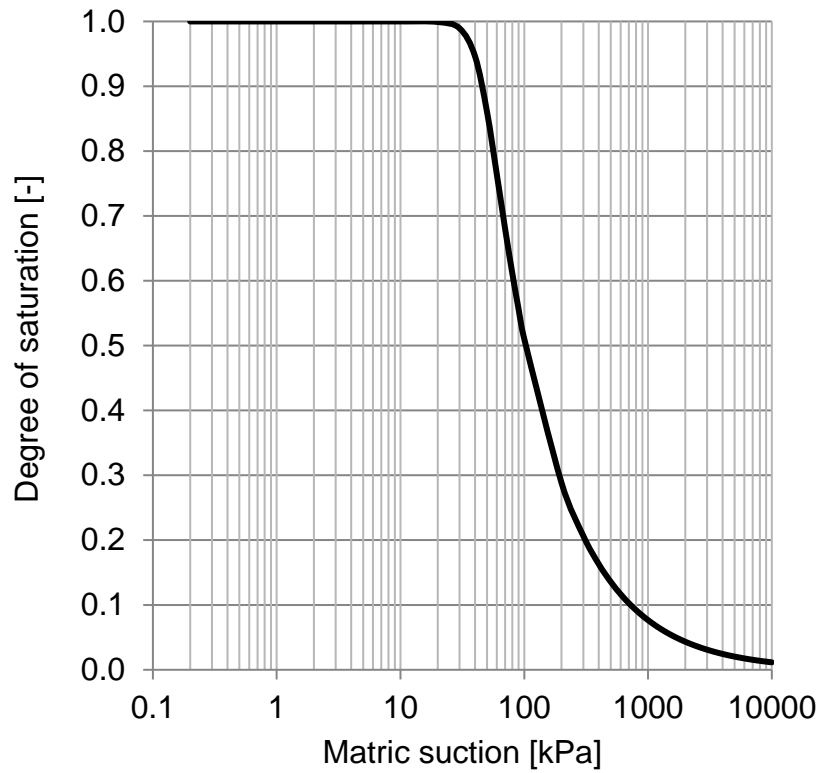
It follows:

$$\varphi' = 32^\circ \quad \text{and} \quad c' = 4.3 \text{ kPa}$$

- c) Thanks to the Gardner's model, the variation of the permeability with respect to the matric suction is drawn in the following figure.



- d) Thanks to the Van Genuchten's model, the variation of the degree of saturation with respect to the matric suction is drawn in the following figure.

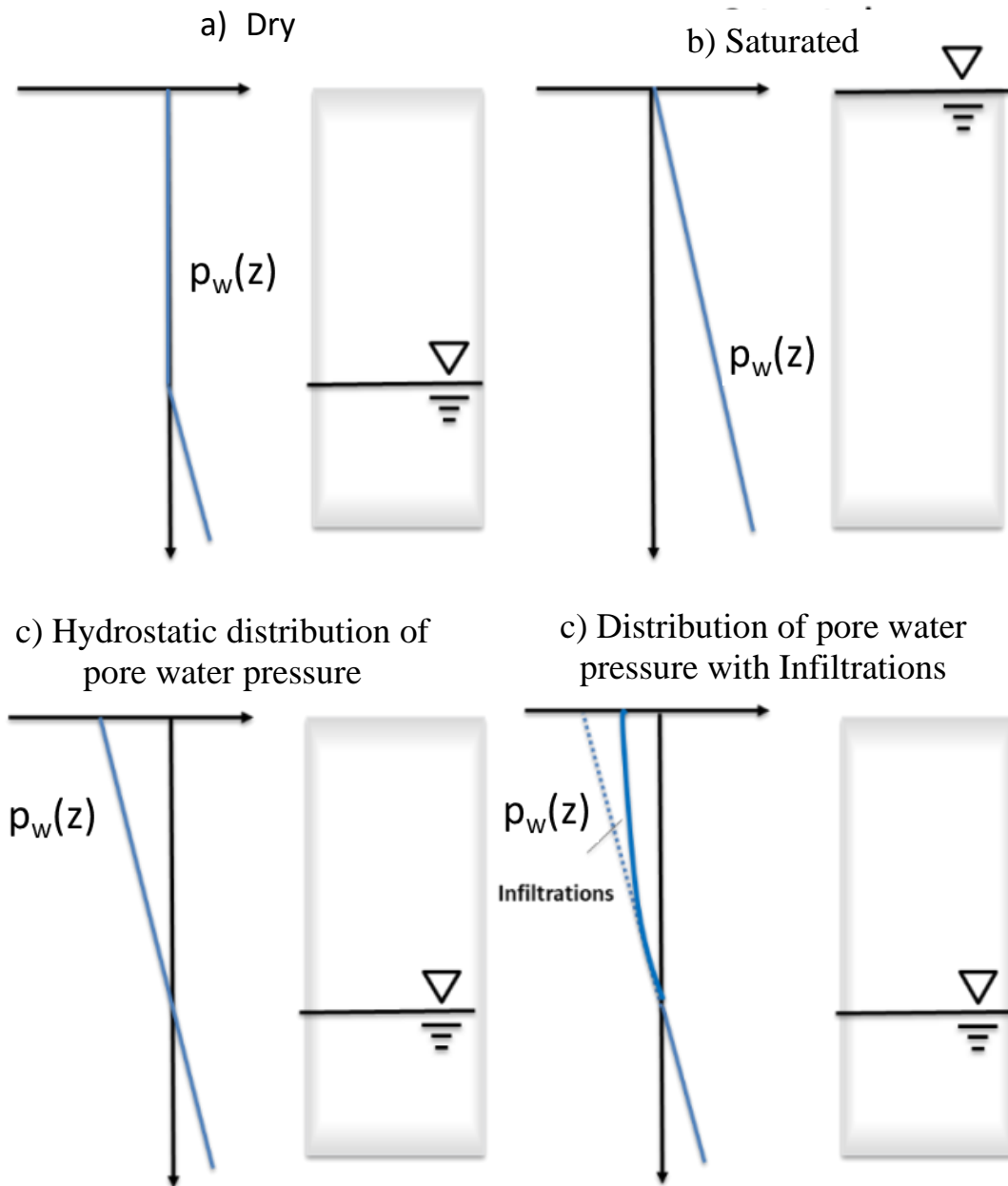


- e) Permeability* can be estimated by finding the corresponding value of matric suction in the chart developed in d), and then finding the corresponding value of permeability in the chart developed in c). $k = 1.0E-07$ m/s. is found when $S_r = 50\%$, while $k = 3.2E-07$ m/s is found when $S_r = 90\%$. In general, permeability of partially saturated soil increases with degree of saturation.

* The terms permeability and hydraulic conductivity are used in the course to indicate the constant of proportionality in Darcy's law. Dimensionally it is LT^{-1} . Intrinsic permeability is measured in L^2 .

2. Lateral Earth Pressure

The water pressure distribution foreseen in the cases of analysis is sketched in the following figure:



The variation of the matric suction over the depth, in the case of a steady state and mono-dimensional flow for a given infiltration rate q , can be determined by using the following formula:

$$(p_a - p_w) = -\frac{1}{\alpha} \ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha (Y-z)} - \frac{q}{k_s} \right]$$

in which α is a model parameter, k_s is the saturated hydraulic conductivity, z is the depth, q is the infiltration rate and γ_w is the unit weight of the water.

By knowing the matric suction at a certain depth z , the degree of saturation can be computed by using the Van Genuchten's model:

$$S_r = \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m$$

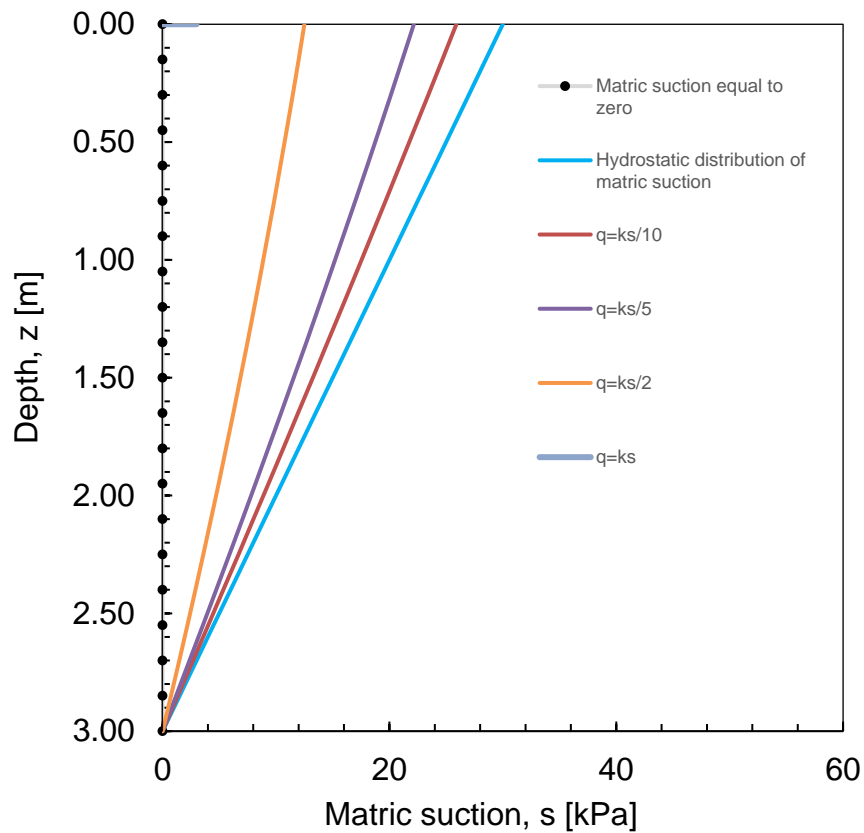
Then, by considering $p_a=0$, the estimation of the lateral earth pressure at a given depth is straightforward thanks to the following formula:

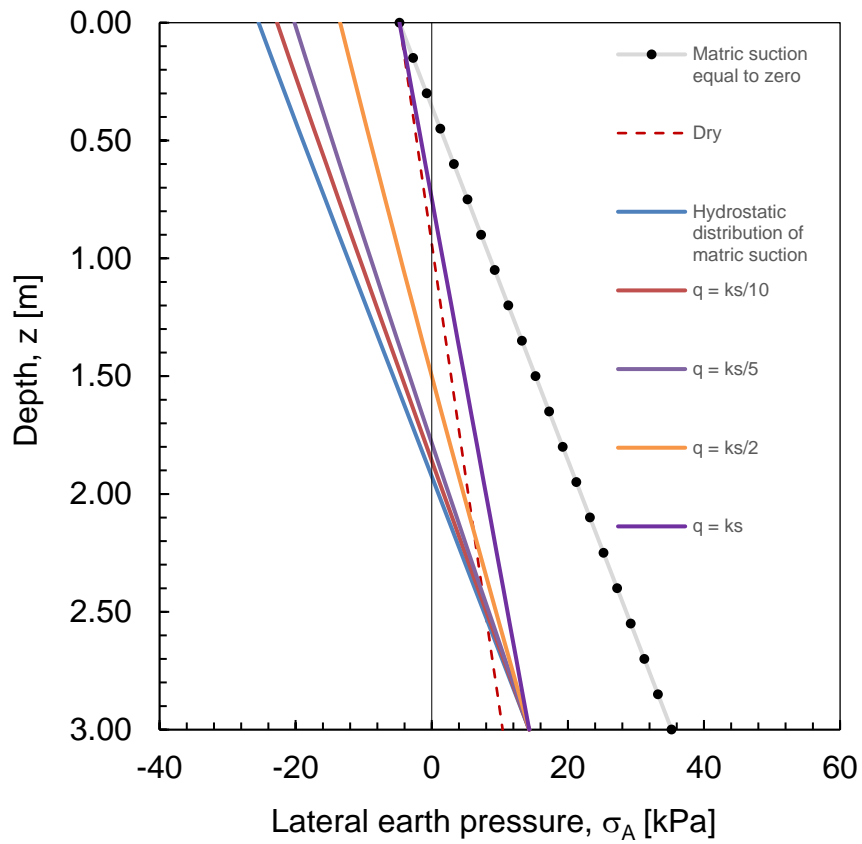
$$\sigma_A = \sigma_v K_A - 2c' \sqrt{K_A} - \left\{ \frac{1}{1 + [\alpha(p_a - p_w)]^n} \right\}^m \left\{ -\frac{1}{\alpha} \ln \left[\left(1 + \frac{q}{k_s} \right) e^{-\gamma_w \alpha z} - \frac{q}{k_s} \right] \right\} (1 - K_A)$$

in which σ_v is the total vertical stress, K_A is the coefficient of lateral stress at active state which can be estimated thanks to the following relation:

$$K_a = \frac{1 - \sin \varphi'}{1 + \sin \varphi'} = \tan^2(45^\circ - \varphi'/2)$$

The distributions over the depth of the matric suction and of the total horizontal stress at the active state for the different cases are shown in the following figures.





According to the lateral earth pressure profile, the resultant horizontal force will increase when a dry soil becomes fully saturated. This can occur during and after a concentrated rainfall event, especially when the drainage system is not efficient.